11/16/17

"The greatest accomplishment is not in never falling, but in rising again after you fall."-Vince Lombardi

HW:

AIM: What is Rectilinear Motion?

Warm Up: Differentiate each of the following with respect to x:

1) $y = e^x + x^{10} - \frac{1}{x}$ $= x^{-1}$

$y' = e^x + 10x^9 + \frac{1}{x^2}$

2) $y = x^2 \ln x$

$y' = 2x \ln x + x^2 \cdot \frac{1}{x}$

$y' = 2x \ln x + x$

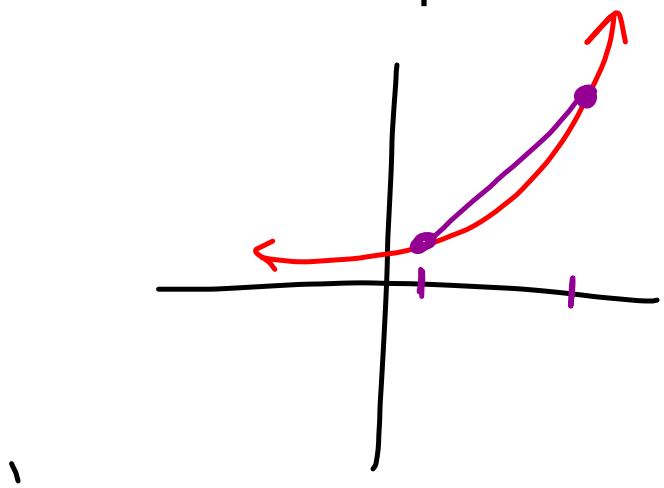
In our study of Calculus, we are often interested when an object (or particle) is speeding up, slowing down, stopped, or has no acceleration.

Recall the notation for the average rate of change (ARoC) of a function $y = f(x)$ over an interval $[x_0 , x_1]$

Δy = change in y

Δx = change in x

REMEMBER the Average Rate of Change is the slope of the secant line.



Instantaneous Velocity:

Tells how fast something is going at that exact instant and in which direction. That is, how the position is changing with respect to time represented by

$$\frac{ds}{dt} = s'(t) = v(t) = \lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t}$$

What is Rectilinear Motion?

* Motion along a straight line

Ex: - A person running
- A car travelling
- A particle moving along a line

Distance:

Position function

$s(t)$

Velocity:

- Rate of change of distance (Position) with direction
- Derivative of Position

$v(t)$

Accelaration:

- Rate of change of velocity with direction
- Derivative of Velocity

$a(t)$

$s(t)$ = Position

$v(t) = s'(t)$ = Velocity

$a(t) = v'(t) = s''(t)$ = Acceleration

1) A particle moves along a horizontal line. Its position is $s(t) = t^3 - t^2 - 56t$ where $t \geq 0$ a "t" represents time.

a) What is the velocity function?

$$s'(t) = v(t) = 3t^2 - 2t - 56$$

b) What is the acceleration function?

$$s''(t) = v'(t) = a(t) = 6t - 2$$

c) What is the particle's position @ $t = 3$?

$$s(3) = 3^3 - 3^2 - 56(3)$$
$$= 27 - 9 - 168$$
$$= \boxed{-150}$$

"−" to the left

"+" to the right

2) Suppose a car is travelling and it's position is $s(t) = t^3 - 6t^2 + 9t + 5$ where $t \geq 0$ and t is in minutes.

a) What is it's position @ $t=0, t=1, t=2, t=3$?

$s(0) = 0^3 - 6(0)^2 + 9(0) + 5 = 5$

$s(1) = 9$

$s(2) = 7$

$s(3) = 5$

b) At what time(s) is the car stopped?

(* Stopped when velocity = 0)

$v(t) = 3t^2 - 12t + 9$

$0 = 3t^2 - 12t + 9$

$0 = 3(t^2 - 4t + 3)$

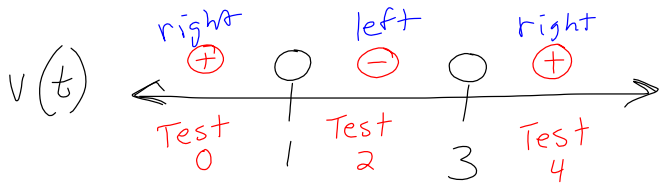
$0 = t^2 - 4t + 3$

$0 = (t-3)(t-1)$

$t = 3 \quad | \quad t = 1$

Stopped @ $t = 1$, $t = 3$

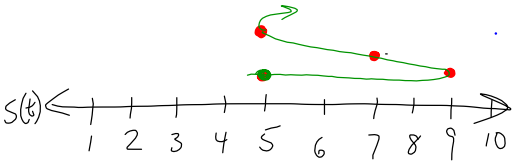
c) When is the car moving left? (Velocity is (−))
When is it moving right? (Velocity is (+))



Recall car stopped @ $t=1, t=3$

$v(0) = 3(0)^2 - 12(0) + 9 = +9$
$v(2) = -3$
$v(4) = +9$

Right:
$[0,1) \cup (3,\infty)$



$t=0$ $t=2$ $t=1$
$t=3$

Left:
$(1,3)$

d) What is the distance travelled in the first 3 minutes?

Look @ the positions and use the intervals where the car is travelling the same direction

$$|s(0)-s(1)| + |s(1)-s(3)|$$

$$|5-9| + |9-5|$$

$$4 + 4$$

$$= 8$$